Linear Algebra I
Revision

Below are some basic definitions one must know, theorems one must be able to prove, and questions one must be able to answer to pass the course. This list is not exhaustive.

Definitions

Definition. Vector space: definition (axiomatic and intuitive/geometric), linear function, dual space, linear dependence, subspace.
Bases: basis, span, dual basis, finitely-generated vector space, dimension.
Linear maps: linear maps, matrices, kernel, image, rank, nullity, columnspace, conjugation.
Eigenspaces: characteristic polynomial, eigenvector, eigenline, eigenvalue, eigenbasis, diagonalisation, similar matrix
Inner product spaces: dot product, inner product, inner product space, angle in inner product spaces, norm, distance, orthogonal

Theorems

Theorem. *The following theorems should be known, without proofs:*

1. *Cayley-Hamilton theorem;*
2. *Norm and distance properties.*

Theorem. *The following theorems should be known, both the statements and proofs:*

1. *two vectors are linearly dependent if and only if they lie on the same ray from the origin;*
2. *every finitely-generated vector space has a basis;*
3. *rank-nullity theorem;*
4. *a matrix of a linear map $L : V \to V$ can be diagonalised if and only if its eigenvectors form a basis of V;*
5. *Cauchy-Schwartz inequality.*

Vector spaces

Question 1. Determine whether a given space is a vector space. Determine whether a given map is a linear map or linear function.

Question 2. Find a basis, dual basis, dual space.

Question 3. Construct a linearly independent set from a linearly dependent set.

Linear maps

Question 4. Write a linear map with respect to a basis (as a matrix). Determine rank, nullity, kernel, and image of a linear map.

Question 5. Convert representations of vectors and linear maps between two bases.

Eigenspaces

Question 6. Find eigenvalues, eigenvectors, eigenlines of a given basis. Determine whether an eigenbasis exists. Diagonalise a given matrix.

Inner product spaces

Question 7. Determine orthogonal complement of a given subspace.

Question 8. Apply the Gram-Schmidt process to form an orthonormal basis from a given basis.